STRUCTURE OF OBLIQUE SHOCK WAVE AT HIGH MACH NUMBERS

Yu. A. Berezin, G. I. Dudnikova, V. G. Eselevich, and R. Kh. Kurtmullaev

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 4, pp. 100–104, 1969

Results are presented of a study of the structure of shock waves propagating at an angle $\pi/2 - \theta$ ($\theta \neq 0$) to a magnetic field as a function of their velocity (Mach number M). These questions for transverse waves have been studied in several works [1–3], in which it was shown that with increase of the Mach number the density and longitudinal velocity profiles in the transverse wave approach the discontinuous form (there is a tendency toward breaking, which is recorded experimentally [4]). The possibility of oblique wave breaking in a cold rarefied plasma has remained obscure since analysis of the corresponding steady-state equations, in contrast with the case of transverse waves [3], does not yield a qualitatively different structure with increase of the Mach number (for the corresponding analysis see below). To clarify this problem computer calculations were made of unsteady plane waves propagating at an angle to an undisturbed magnetic field and the results were compared with experiment. It was found that with increase of the wave amplitude the oscillatory precursor typical of the quasi-stationary wave disappears. For values $M_* \approx 5-6$ the magnetic field profile takes on an aperiodic nature, and the particle velocity profile approaches the discontinuous profile. We observe experimentally the formation of a trough in the front and its broadening, similar to the way this takes place in the normal wave.

The most general approach to rarefied plasma dynamics problems lies in use of the kinetic equations for the electrons and ions with self-consistent electromagnetic fields. However, this technique is very complicated and in many cases is not necessary. The hydrodynamic approximation is often quite adequate for many problems, i.e., the solution of the moment equations (macroscopic equations of motion of the electrons and ions) and the Maxwell equations with self-consistent electromagnetic fields. Many questions associated with the propagation of waves of finite amplitude and shock waves in a rarefied plasma have been studied within the framework of this approximation [3, 5].

1. Mathematical formulation of the problem. Consider the following problem. At the initial time in the uniform magnetic field H_0 there is a quasi-neutral cold plasma (i.e., the initial pressure $P_0 \ll H_0^2/8\pi$) with density N_0 and transverse dimension R. Then a magnetic field directed at the angle θ to the field H_0 begins to grow sinusoidally at the plasma-vacuum boundary. The plasma volume is compressed, and plane disturbances propagate ahead of the piston at the angle $\pi/2 - \theta$ to the field H_0.

The system of equations corresponding to the problem of the propagation of these disturbances has the form

$$
\begin{gathered}
\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}(N u_x) = 0 \\
\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} = -\frac{1}{8\pi m_i N}\frac{\partial}{\partial x}(H_y{}^2 + H_z{}^2) \\
\frac{\partial u_{yz}}{\partial t} + u_x \frac{\partial u_{y,z}}{\partial x} = \frac{H_x}{4\pi m_i N}\frac{\partial H_{yz}}{\partial x} \\
\frac{\partial H_y}{\partial t} = -\frac{\partial}{\partial x}(u_x H_y - u_y H_x) + \frac{cH_x}{4\pi e}\frac{\partial}{\partial x}\left(\frac{1}{N}\frac{\partial H_z}{\partial x}\right) \\
+ \frac{m_e c^2}{4\pi e^2}\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t} + u_x \frac{\partial}{\partial x}\right)\left(\frac{1}{N}\frac{\partial H_y}{\partial x}\right) + \frac{c^2}{4\pi}\frac{\partial^2}{\partial x^2}\left(\frac{H_y}{\sigma}\right) \\
\sigma = \frac{Ne^2}{m_e \nu} \\
\frac{\partial H_z}{\partial t} = -\frac{\partial}{\partial x}(u_x H_z - u_z H_x - \frac{cH_x}{4\pi e}\frac{\partial}{\partial x}\left(\frac{1}{N}\frac{\partial N_y}{\partial x}\right) + \\
+ \frac{m_e c^2}{4\pi e^2}\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t} + u_x \frac{\partial}{\partial x}\right)\left(\frac{1}{N}\frac{\partial H_z}{\partial x}\right) + \frac{c^2}{4\pi}\frac{\partial^2}{\partial x^2}\left(\frac{H_z}{\sigma}\right) \qquad (1.1) \\
H_x = \mathrm{const}
\end{gathered}
$$

Here σ is the effective conductivity, ν is the effective collision frequency, u is the shock wave velocity, m_e is the electron mass, m_i is the ion mass, c is the speed of light, and e is the electron charge.

The boundary conditions are

$$H_z(0, t) = H_0(\cos\theta + A\sin\omega t), \quad H_y(0, t) = 0, \ A = H_\infty^\circ/H_0 \tag{1.2}$$

Here H_∞° is the amplitude of the external field, ω is the external field variation frequency, and θ is the angle between the plane of the front and the direction of the unperturbed magnetic field. A similar system of equations without dissipation and for comparatively low Mach numbers was studied in [5, 6], where an oscillatory structure of the oblique waves was obtained and the question of the reflection of such waves from the axis of the system was studied. In the present study problem (1.1), (1.2) was solved numerically for conditions quite close to those of the experiments of [7].

2. **Nature of the stationary solutions.** To clarify the question of the nature of the stationary solutions of (1.1) we examine the type of singular points corresponding to the disturbed (behind the wave) and undisturbed (ahead of the wave) plasma states, similar to the way this was done in [3] for the case of transverse shock waves. Linearizing the system of stationary equations obtained from (1.1), we find the characteristic equation

$$C^2K^2 - \frac{H_0^2}{4\pi\rho_0}\eta K - B = 0 \tag{2.1}$$

where

$$C = \frac{cH_x}{4\pi e N_0}, \quad \eta = \frac{c^2}{4\pi\delta}, \quad B = \left(\frac{H_x^2}{4\pi\rho_0 u} - u\right)\left(u - \frac{H_0^2}{4\pi\rho_0 u}\right), \quad \rho_0 = N_0 m_i$$

The roots of the characteristic equation have the form

$$K_{1,2} = \frac{H_0^2}{8\pi\rho_0 c^2}\eta \pm i\frac{\sqrt{|B|}}{C} \tag{2.2}$$

since the shock wave is supersonic relative to the undisturbed state. According to [8] a singular point with roots of the (2.2) type is a focus. It follows from (2.2) that its nature is independent of the wave amplitude.

Linearization of the stationary equations near the disturbed state leads to a characteristic equation whose roots are real and have different signs (in view of the fact that the shock wave is subsonic relative to the disturbed state) regardless of the wave amplitude. According to [8] such a singular point is a saddle.

Thus, analysis of the stationary states shows that regardless of the oblique shock wave amplitude the transition belongs to the saddle-focus type, i. e., it is continuous. We recall that in the transverse wave case the transition for comparatively small amplitudes will be continuous (of the saddle-focus type) while for large amplitudes it will be discontinuous (of the saddle-saddle type) [3].

However, the preliminary experiments showed that for large Mach numbers the structure of the oblique shock waves experiences changes which can be interpreted as breaking. In this connection the solution of the unsteady problem of strong oblique wave propagation becomes particularly important.

3. **Numerical solution results.** Let us examine the results of the numerical solution of (1.1), (1.2). In the small dissipation case

$$\nu \sim (10^{-2} - 10^{-1})\,\omega_{H_e}, \qquad \omega_{H_e} = \frac{eH_0}{m_e c}$$

for low Mach numbers

$$M = \frac{u}{V_A} \lesssim 2, \quad V_A = \frac{H_0}{\sqrt{4\pi N_0 m_i}}$$

(where V_A is the Alfven velocity), a quasi-steady shock wave is formed with an oscillatory precursor (Fig. 1). The characteristic scales of the oscillation δ and oscillatory trail precursor length

$$\delta \sim (0.5 - 1)\frac{c}{\omega_{0i}}, \qquad \Delta \sim (2 - 2.5)\frac{c}{\omega_{0i}}$$

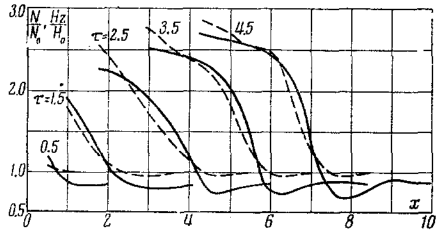

Fig. 1. Profile of "oblique" shock wave for M < M_* at successive instants;

$$\varkappa = \frac{\nu}{\omega_{H_e}} = 0.2, \quad A = \frac{H_\infty^0}{H_0} = 2$$

$$\tau = \frac{V_A t \omega_{0i}}{c}, \quad x = \frac{r\omega_{0i}}{c}$$

are, respectively, the collision frequency, maximum piston amplitude, time, and distance in relative units; θ = 30°; the dashed curve is the plasma density, the solid curve is the magnetic field.

agree well with the theoretical values [9]

$$\delta \sim \frac{c}{\omega_{0i}}\Theta, \qquad \Delta \sim \frac{u}{v}\frac{m_i}{m_e}\left(\Theta^2 + \frac{m_e}{m_i}\right)$$

Here ω_{0i} is the ion Langmuir frequency.

The magnetic field profile leads the particle density profile (Fig. 1) by the distance

$$l \sim 0.2\frac{c}{\omega_{0i}} \sim \frac{c^2}{4\pi\sigma V_A(M-1)} < \frac{c}{\omega_{0i}}\Theta$$

which is consistent with the resistive dissipation mechanism in the front. With increase of the dissipation there is a continuous change of the shock wave profile from oscillatory to monotonic with the oscillations disappearing when

$$\sigma \lesssim \frac{\omega_{0i}}{4\pi}\frac{c}{u\Theta}$$

Increase of the magnetic field amplitude leads to marked unsteadiness of the process and a significant change of the shock wave structure. These changes show up in the following:

1) in the course of time the previously formed magnetic field oscillations disappear (the front structure becomes aperiodic); in this process some broadening of the main magnetic field shock is observed (Fig. 2);

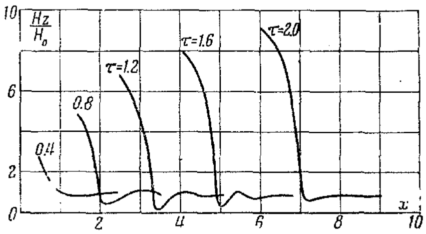

Fig. 2. Transformation of oblique shock wave magnetic profile near the critical Mach number M ($A = 10, \varkappa = 0.2$, $\theta = 30°$).

2) the steepness of the particle density and longitudinal velocity profiles increases sharply, i. e., Δ_N, $\Delta_{u_x} \to 0$ (Fig. 3), while the width Δ_H of the magnetic field profile remains practically constant (isomagnetic shock).

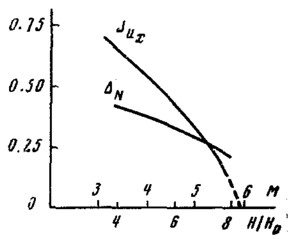

Fig. 3. Mach number dependence of the width Δ_{u_x} of the u_x velocity profile and the density jump ΔN in the oblique shock wave front with approach to $M_* \sim 5-6$ ($A = 10$, $\varkappa = 0.2$).

These results indicate approach of the oblique shock wave at large amplitudes to the "breaking" phase. For the case $\Theta \approx 30°$ the value of the Mach number for which the breaking tendency is observed is $M_* \sim 5-6$ and the value of the relative wave amplitude is $h_* = H/H_0 \sim 7-8$.

4. **Experimental results.** The experiments were conducted on a UN-4 setup [2, 7]. The shock waves were generated in a cylindrical plasma volume (diameter d = 16 cm, length l = 100 cm) with the aid of a "magnetic piston" excited by a shock coil. The variable field of the coil ($H_\infty^\circ \approx$ 2kOe) is nonuniform along its edge; therefore, the piston pressure drops off rapidly with increasing distance from the edge of the coil. As a result the direction of piston motion relative to the system axis forms an angle $\pi/2 - \theta$. In the presence of an initial magnetic field H_0 ($H_0 \approx 100$

−1000 kOe parallel to the axis, this leads to the excitation of an "oblique" wave, a typical profile of which is shown in Fig. 4.

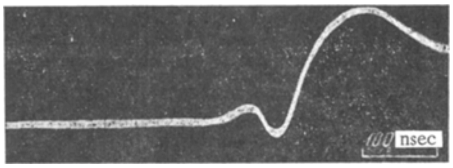

Fig. 4. Typical oblique shock wave profile in hydrogen $N_0 \sim 5 \cdot 10^{13}$ cm^{-3}, $\theta \sim 15°$.

The measurements were made using magnetic probes located in the r, z plane, which made it possible in each experiment to record directly the magnetic field H(r, z, t), the wave velocity components u_r, u_z, and the front slope θ relative to the direction of the initial magnetic field H_0. Specifically, the magnetic disturbances inside and outside the coil were recorded simultaneously for comparison of the transient processes in the normal and oblique waves. The inner probe 1 was located at the distance $r_1 = 2$ cm from the system axis near the edge of the coil, where the piston travels across the field ($\theta \approx 0°$). The outer probe 2 was located at the same radius ($r_2 = 2$ cm) and at the distance $\Delta Z = 5$ cm from the first probe. In this region the piston motion takes place at an angle $\pi/2 - \Theta$ to the field H_0, where $\Theta \gg \sqrt{m_e/m_i}$.

Figure 5 shows a sequence of oscillograms taken from these probes with reduction of the initial field H_0 from 450 to 100 Oe in hydrogen plasma with density $N_0 \sim 5 \cdot 10^{13}$ cm^{-3}. For a constant amplitude of the "magnetic piston" this leads to increase of the relative wave amplitude and Mach number M. The average front slope Θ in the region near probe 2 was determined from the time shift Δt between the magnetic signals of the first and second probes (Fig. 5, b–c, d–e, f–g, h–i) using the relation $\Theta \approx \arcsin(U\Delta t/\Delta Z)$, where u is the velocity of the oblique shock wave and $\Delta Z = 5$ cm is the distance between the first and second probes.

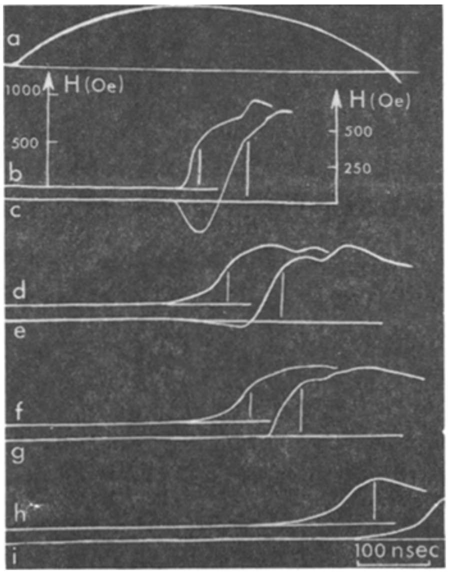

Fig. 5. Modification of magnetic front structure with change of the relative amplitude of the normal (b, d, f, h, $\theta \approx 0°$) and oblique (c, e, g, i, $\theta \sim 15$–$30°$) shock waves; hydrogen; $N_0 \sim 5 \cdot 10^{13}$ cm^{-3}; a) variation of external field $H_\infty(t)$. The magnetic field scales on oscillograms d, f, h and e, g, i are, respectively, equal to the scales shown for the signals b and c. The initial magnetic field $H_0 \sim 450$ Oe (b, c), 200 Oe (d, e), 150 Oe (f, g), 100 Oe (h, i).

We see from Fig. 5b, d, e, h that with increase of the relative amplitude h there is a restructuring of the front of the normal wave as described in [4], the formation and growth of the trough, and broadening of the front to the value $\Delta \sim c/\omega_{0i}$.

The oblique wave magnetic field profile also experiences restructuring, which takes place with some lag relative to the transient processes in the normal wave, explained in part by the smaller value of h in the region outside the coil (because of the weakening of H°_∞ at the edge of the coil).

An essential characteristic of these processes in the oblique shock wave front is the gradual disappearance of the oscillation preceding the main shock, as a result of which the profile takes on an aperiodic nature (Fig. 5c, e, g). Up until this instant the basic features of the wave structure change are modeled fairly well in the computer experiment results presented above (Fig. 2).

Further increase of the relative amplitude h in the oblique wave leads to the formation of a trough and broadening of the front (Fig. 5i).

In this phase the magnetic field profiles of the normal ($\Theta \approx 0°$) and oblique ($\Theta \approx 30°$) shock waves are practically indistinguishable (Fig. 5h, i).

The described phenomenon develops at Mach numbers $M \sim 3-5$ (later results of measurements of the electric potential φ have shown that the disturbance passes through two critical states: formation of N, φ shock with width on the order of the Debye radius r_D ($M \sim M_{*1}$) and "breaking" of this wave ($M \gtrsim {}_{*2}$)).

We note that with reduction of the quasi-stationary field H_0 (i. e., with increase of h) the angle Θ increases, approaching for $H_0/H^\circ_\infty \ll 1$ the value determined by the curvature of the lines of force of the external field $H_\infty(t)$ in a vacuum (initial pressure $P_0 \ll H_0^2/8\pi$). However, this change of the angle only leads to satisfaction of the fundamental condition of the present experiment: $\Theta \gg \sqrt{m_e/m_i}$. Simultaneous measurements made of the plasma density N and magnetic field H showed the occurrence of N shocks near $M = M_*$, which was also confirmed by the results of the computer solution.

Thus the experimental and numerical results indicate modification of the structure of the strong oblique shock waves which can be interpreted as breaking.

In conclusion the author wishes to thank R. Z. Sagdeev for helpful discussions and assistance in this study.

REFERENCES

1. R. Z. Sagdeev, "Collective processes and shock waves in rarefied plasma," collection: Plasma Theory Problems [in Russian], no. 4, Atomizdat, Moscow, 1964.

2. R. Kh. Kurtmullaev, Yu. E. Nesterikhin, V. I. Pil'skii, and R. Z. Sagdeev, "Plasma heating by collisionless shock waves" [in Russian], Plasma Physics and Controlled Nuclear Fusion: Proc. Conf. Culham, 1965, Vienna, vol. 2, 1966.

3. Yu. A. Berezin and R. Kh. Kurtmullaev, "Cylindrical waves in a rarefied plasma in the presence of strong collisionless dissipation," Fizika goreniya i vzryva [Combustion, Explosion, and Shock Waves], vol. 2, no. 3, 1966.

4. R. Kh. Kurtmullaev, V. L. Masalov, K. I. Mekler, and V. I. Pil'skii, Pis'ma v redaktsiyu ZhETF, vol. 7, no. 2, 1968.

5. K. W. Morton, "Finite-amplitude compression waves in a collision-free plasma," Phys. Fluids, vol. 7, no. 11, 1964.

6. Yu. A. Berezin, "Cylindrical waves of finite amplitude in a rarefied plasma," PMTF [Journal of Applied Mechanics and Technical Physics], No. 5, 1965.

7. V. G. Eselevich, R. Kh. Kurtmullaev, and V. I. Pil'skii, "Influence of dispersion effects on shock wave structure in magnetized plasma," Pis'ma v redaktsiyu ZhÉTF, vol. 5, no. 9, 1967.

8. V. V. Nemytskii and V. V. Stepanov, Qualitative Theory of Differential Equations [in Russian], Gostekhizdat, Moscow-Leningrad, 1947.

9. Yu. A. Berezin, R. Kh. Kurtmullaev, and Yu. E. Nesterikhin, "Collisionless shock waves in rarefied plasma," Fizika goreniya i vzryva, vol. 2, no. 1, 1966.

13 January 1969

Novosibirsk